thrust-to-weight ratio on the relative severity of in-flight beam columning action. For most contemporary liquid boosters with a T/W ratio of 1.25, Eq. (13) would indicate that the free-free beam columning increase is only one-half that of the fixed-free condition. This has been found to be the case in a large booster, which experiences a 13% increase on the launch pad (with $K_\phi = \infty$) and a 6 to 7% increase in flight. As the thrust-to-weight ratio increases, the relative severity of the flight beam columning becomes more prominent.

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Wave Reflection from the Intersection of Oblique Shock Waves of the Same Family

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Nomenclature

M = Mach number
p = static pressure
γ = specific heat ratio
δ = flow deflection angle for oblique shock
θ = flow angle
σ = oblique shock angle

Subscripts

1, 2, 3, 4 = regions defined by Fig. 1

When oblique plane shock waves intersect and coalesce, as shown in Fig. 1, a "three-shock system" results. At the intersection of the shocks, a slip line is produced, and a weak wave may be propagated into the two-shock side (region 3). This reflected wave can be either expansive or compressive, depending upon the combination of M_1, δ_1, and δ_2.

To determine which type of wave will be reflected, an investigation was carried out for two equal turn angles, $\delta_1 = \delta_2 = \delta$, and various Mach numbers, M_1, for perfect gas γ values of 1.28, 1.40, and 1.67. This was performed by digital computer solution of the "nonreflection" case, i.e., the combination of M_1 and δ which gives a zero-strength reflected wave. The requirement for this is that $p_3 = p_4$ and $\theta_3 = \theta_4$.

The oblique shock relations are

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 \sin^2\sigma_1 - (\gamma - 1)}{\gamma + 1} \tag{1}$$

$$M_2^2 = \frac{(\gamma + 1)^2 M_1^4 \sin^2\sigma_1 - 4(M_1^2 \sin^2\sigma_1 - 1)(\gamma M_1^2 \sin^2\sigma_1 + 1)}{[2\gamma M_1^2 \sin^2\sigma_1 - (\gamma - 1)][(\gamma - 1)M_1^2 \sin^2\sigma_1 + 2]} \tag{2}$$

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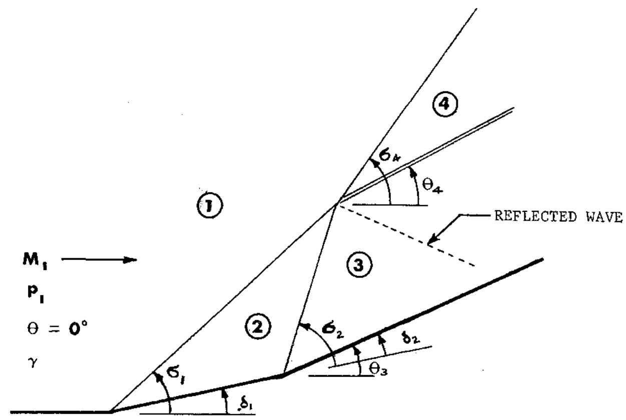

Fig. 1 Model for the three-shock system.

For the relation between turning angle δ and shock angle α, Thompson's cubic equation in $\sin^2\alpha$ is used[1]:

$$\sin^6\sigma_1 + b \sin^4\alpha_1 + c \sin^2\sigma_1 + d = 0 \tag{3}$$

where

$$b = -[(M_1^2 + 2)/M_1^2] - \gamma \sin^2\delta_1$$

$$c = \frac{2M_1^2 + 1}{M_1^4} + \left[\frac{(\gamma + 1)^2}{4} + \frac{(\gamma - 1)}{M_1^2}\right] \sin^2\delta_1$$

$$d = -\cos^2\delta_1/M_1^4$$

Similar relations, of course, exist for the shocks between regions 2 and 3 and regions 1 and 4. For the cubic equation, three real roots exist for $\sin^2\sigma$. The smallest results in an expansion shock, violating the second law of thermodynamics, and must be disregarded. The largest root corresponds to the strong shock solution, which will not exist as a plane shock for such a geometry. The middle value is the root corresponding to the weak shock and is the one having physical significance.

A Fortran program was prepared for the IBM 1620 computer to find by interaction the "no-reflection" condition. The sign of the derivative $(d/dM_1)/(p_3/p_4)$ was determined by the computer. If the derivative has a positive sign, higher

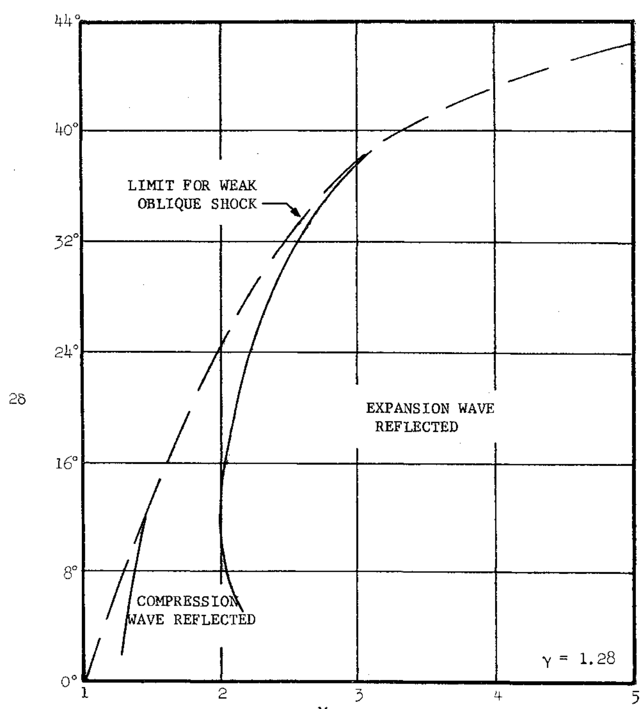

Fig. 2 Conditions for no-reflected wave from the shock intersection ($\gamma = 1.28$).

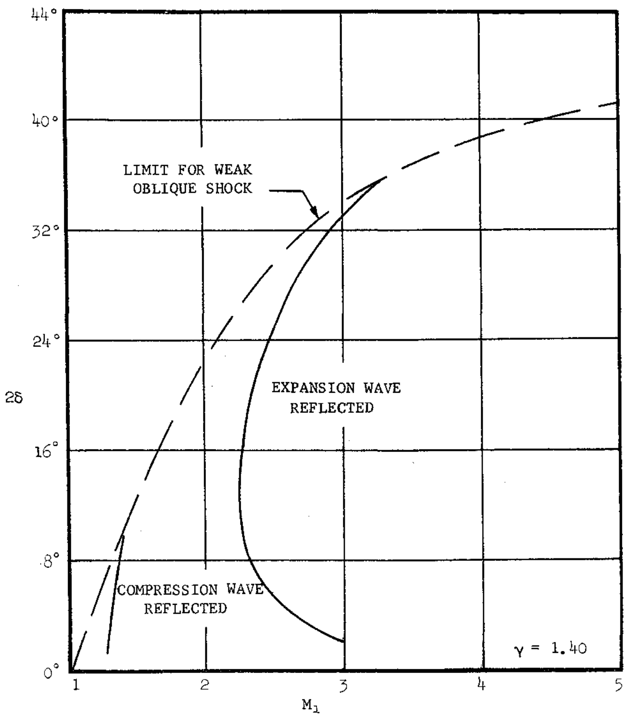

Fig. 3 Condition for no-reflected wave from the shock intersection (γ = 1.40).

values of M_1 will produce a reflected expansion wave, and lower values of M_1 will produce a reflected compression wave. The inverse obviously holds for negative signs.

Figures 2–4 show as solid lines the resulting no-reflection conditions and the regions of each type of reflected wave. It may be seen that two values of M_1 meet the no-reflection requirements at low total turning angles. Only one solution exists at moderate angles, and no solution exists for high angles. Likewise, for a given value of M_1, either two, one, or no values of total turn angle may meet this requirement. Changes in γ give no qualitative change in the result.

The numerical data from which the graphs are plotted are available in the M. S. report of the junior author[2] and are obtainable from the School of Mechanical Engineering, Oklahoma State University.

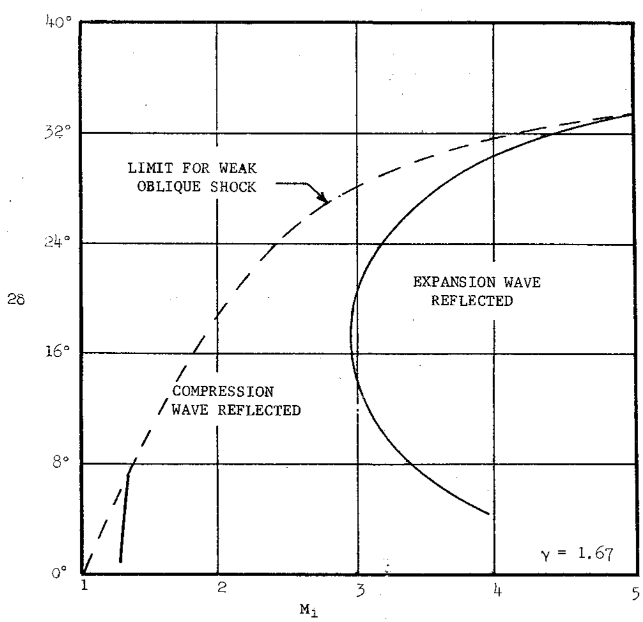

Fig. 4 Condition for no-reflected wave from the shock intersection (γ = 1.67).

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Recombination Losses in Rocket Nozzles with Storable Propellants

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THE advent of space rocket engines using low chamber pressures and high area ratios has focused attention on the loss in I_{sp} due to incomplete recombination of free radical species during nozzle expansion. Analyses are available[1–3] to determine the losses once the chemical kinetics and their reaction rate constants are known. The kinetics for propellant combinations such as hydrogen-oxygen[2] and RP-1 oxygen[4,5] are fairly well known. However, recombination data for storable propellant combinations such as nitrogen tetroxide-mixed amines have not been available. This note presents a method of accounting for recombination losses for such storable combinations.

Inspection of the species present in rocket combustion chambers, shown in Table 1, using the propellants nitrogen tetroxide in a 50-50 by weight mixture of unsymmetrical dimethyl hydrazine and hydrazine reveals the following facts:

1) The free radical or "active" species present which can recombine in the nozzle to yield thermal energy include CO, H, OH, NO, and O.

2) The number of moles of these species decrease with increasing pressure. The decrease is especially rapid for the species O, OH, and H.

3) The number of moles of molecular nitrogen is large. The amount of nitrogen in other nitrogen-containing fragments is quite small. Thus, nitrogen reactions will not affect the I_{sp} recoverable in the nozzles due to the relatively small number of moles of non-N_2-nitrogen species and the fact that the energy levels of their recombination reactions are low.[6]

Therefore, the main heat-producing reactions in the nozzle are the following:

$$H + H + M \rightleftharpoons H_2 + M$$

$$OH + H + M \rightleftharpoons H_2O + M$$

$$O + O + M \rightleftharpoons O_2 + M$$

$$CO + O + M \rightleftharpoons CO_2 + M$$

These species are the same as those present in the combustion of RP-1-oxygen and hydrogen-oxygen. The chemical kinetics for these reactions are fairly well known. Thus, the kinetics of the C-H-O system can be used in the case of combustion of nitrogen tetroxide-amine combinations.

An approximate analysis has been developed by Kushida[2] and Koppang et al.[4] using the freezing-point criterion of Bray for the C-H-O system. Kinetic principles allow one to use the freeze-point relaxation criteria of Bray for atomic recombination and one-dimensional shifting equilibrium data to estimate nonequilibrium performance of the forementioned propellants with the following assumptions:

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